Control strategy for fixed-time leader–follower consensus for multi-agent systems with chained-form dynamics

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Control strategy for fixed-time leader–follower consensus for multi-agent systems with chained-form dynamics

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Abstract This paper is concerned with the fixed-time consensus problem of multiple chained-form systems under matched perturbations. In this study, the leader (which can be dynamic) only transmits its state and control input to its neighbors. For each agent, a decentralized observer is designed to estimate the leader state in a fixed-time. Contrary to finite-time schemes, the estimation of the settling time does not require the knowledge of the initial state, allowing a step-by-step design for the controller. A decentralized observer-based control protocol is proposed for each agent to solve the leader–follower consensus problem in a fixed-time. This paper ends with a numerical example showing the effectiveness of the proposed approach.

Keywords Multi-agent system · Fixed-time stability · Chained-form dynamics · Tracking consensus · Decentralized observer · Nonlinear systems · Lyapunov stability

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1 Introduction

In recent years, many researchers have worked on cooperative control of multi-agent systems (MAS) because of its variety of applications in several areas, e.g., target tracking [1], flocking [2], swarming [3], rendezvous [4], monitoring [5], formation control [6,7], etc. In these works, it has been shown that the use of multiple agents improves performances compared to a single one.

From all the applications mentioned above, one basic research topic in MAS is consensus, whose goal is to design control protocols that allow agents to reach an agreement with a certain amount of interest through local interaction [8,9]. Many consensus protocols have been proposed when there is no leader or when the leader is static [10,11]. Nevertheless, it is obvious that a leader (virtual or not) whose behavior is independent of the other agents is required for many applications (e.g., monitoring, formation control, etc.).

The leader–follower consensus problem also referred as consensus tracking problem has been firstly introduced in [12]. In [13], some decentralized tracking control schemes have been developed for second-order MAS considering directed interconnection topology. A consensus tracking protocol has been proposed in [14] for second-order MAS with matched perturbations. In

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[15], the leader–follower consensus problem for linear MAS has been studied using decentralized impulsive control.

It is clear that for the cooperative control of MAS, one of the most important performance indexes is the convergence rate. In fact, it is a significant criterion which describes the effectiveness of the control algorithms. In [16], it has been shown that for MAS represented by a simple integrator, the algebraic connectivity, that is to say, the smallest eigenvalue of the Laplacian graph, determines the convergence time. In [17], the authors have proposed an approach to increase this algebraic connectivity. However, linear algorithms have only focused on asymptotic convergence, where the time to reach consensus can be arbitrarily large. Nevertheless, in some practical cases, finite-time convergence is very interesting in terms of accuracy (which depends on the sampling period) and robustness against perturbations. In [18], the authors have introduced a terminal-sliding mode controller to deal with the finite-time consensus problem of second-order linear uncertain systems. In [19] the finite-time tracking problem has been addressed for second-order MAS. Some sliding mode controllers have been introduced for the tracking control problem of one agent in chained-form dynamics in finite-time [20], in fixed-time [21] and in predefined-time [22]. In [23], a decentralized finite-time controller has been proposed for a group of nonlinear agents with strict-feedback structure. In [24,25], some finite-time agreement protocols for multiple nonholonomic MAS have been given. In [24], an interesting decentralized finite-time convergent observer has been proposed for each agent to estimate the state of the leader in finite-time. However, it should be highlighted that in these studies, the estimated bound of the settling time depends on the initial states of all the agents. Therefore, this bound cannot be a priori estimated in decentralized architectures since the initial state of all agents is not known by all agents.

In this paper, the concept of fixed-time stability is used. This concept has been introduced to design controller such that the convergence time is upper bounded independently of the initial conditions of the system [26–28]. Fixed-time consensus protocols have been investigated for first-order switched MAS with continuous-time and discrete-time subsystems in [29], for first-order MAS with external perturbations in [30–32], second-order dynamics in [33–35] and high-order linear dynamics in [36–38]. Few consensus protocols consider nonlinear MAS (such as chained-form dynamics), which can model dynamics of robots. Furthermore, most of these controllers are designed without ensuring a fixed-time stabilization. Recently, a switching strategy has been introduced to deal with the fixed-time consensus problem for multiple nonholonomic agents [11]. However, in this work, the leader was static and no uncertainty was considered.

In this paper, a new leader–follower consensus protocol for MAS with chained-form dynamics is introduced. The main contributions of the proposed scheme can be summarized as follows:

(i) To remove the problem of the communication loop due to the dependence of the control inputs of the followers on the inputs of its neighbors in [11,33], decentralized observers are designed to estimate the leader state in a prescribed time which does not depend on the initial state contrary to existing finite-time observers,
(ii) a new decentralized observer-based control protocol is proposed for each follower to solve the fixed-time consensus tracking problem when the leader is dynamic,
(iii) robustness properties against matched perturbations is guaranteed,
(iv) an upper bound of the settling time, which only depends on the controller parameters is estimated independently of the initial conditions, contrary to existing finite-time controllers.

This paper is organized as follows. In Sect. 2, some concepts on fixed-time and graph theory are briefly reviewed and the fixed-time leader–follower consensus problem for MAS with chained-form dynamics is presented. In Sect. 3, the decentralized fixed-time observers and the decentralized observer-based controller are derived to solve this problem. Simulations results illustrate the effectiveness and the robustness of the proposed scheme in Sect. 4. Finally, conclusions are given in Sect. 5.

Notations $\lambda_{\min}(P)$ and $\lambda_{\max}(P)$ represent, respectively, the smallest and largest eigenvalue of a square matrix P. Matrix $\text{diag}(a_1, a_2 \ldots a_{N-1}, a_N)$ denotes the corresponding diagonal matrix. For $b \geq 0$, function $\lfloor . \rceil^b$ is defined as $\lfloor \xi \rceil^b = \left(\text{sign}(\xi_1)|\xi_1|^b, \ldots, \text{sign}(\xi_N)|\xi_N|^b\right)^T$ with $\xi = (\xi_1, \ldots, \xi_N)^T \in \mathbb{R}^N$.

2 Problem statement and preliminaries

In this section, some useful concepts on fixed-time stability and graph theory are recalled. Then, the leader–follower consensus problem for MAS with chained-form dynamics is formulated.

2.1 Recalls on fixed-time stability

Considering system

$$\dot{\xi}(t) = g(t, \xi(t))$$
$$\xi(0) = \xi_0 \tag{1}$$

where $\xi \in \mathbb{R}^n$ is the state, $g : \mathbb{R}^+ \times \mathbb{R}^n \to \mathbb{R}^n$ is a nonlinear function and $f(t, 0) = 0$ for $t > 0$. Here, it should be highlighted that solutions of (1) are understood in the Filippov sense [39].

Definition 1 [27] The origin of system (1) is a globally fixed-time equilibrium if it is globally asymptotically stable and there is a positive constant $T_{\max} > 0$ such that for all $\xi_0 \in \mathbb{R}^n$, the solution $\xi(t, \xi_0)$ of system (1) is defined and $\xi(t, \xi_0) \in \mathbb{R}^n$ for $t \in [0, T_{\max})$ and for all $t \geq T_{\max}$, $\xi(t, \xi_0) = 0$.

Lemma 1 [27] *Suppose there is a candidate Lyapunov function* $V : \mathbb{R}^n \to \mathbb{R}^+$ *such that*

$$\dot{V}(\xi) \leq -(\rho V^p(\xi) + \sigma V^q(\xi))^k \tag{2}$$

with $\rho, \sigma, p, q, k > 0$, $pk < 1$ *and* $qk > 1$. *Then, the origin of system* (1) *is globally fixed-time stable with*

$$T_{\max} = \frac{1}{\rho^k(1 - pk)} + \frac{1}{\sigma^k(qk - 1)} \tag{3}$$

Remark 1 [40] If $p = 1 - \frac{1}{\mu}$ and $q = 1 + \frac{1}{\mu}$ with $\mu > 1$, the bound of the settling time can be estimated by a less conservative bound

$$T_{\max} = \frac{\pi\mu}{2\sqrt{\rho\sigma}} \tag{4}$$

2.2 Recalls on graph theory

Let us consider a group of $N + 1$ agents with one leader and N followers. Among the N followers, the communication topology can be represented by graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$ where $\mathcal{V} = \{1, \ldots, N\}$ defines the set of nodes, corresponding to the followers, and $\mathcal{E} \subseteq \{\mathcal{V} \times \mathcal{V}\}$ defines the edge set. A link $(j, i) \in \mathcal{E}$, with $i \neq j$, exists if agent i receive information from its neighbor j. The adjacency matrix $A = (a_{ij}) \in \mathbb{R}^{N \times N}$ satisfies $a_{ij} > 0$ if $(j, i) \in \mathcal{E}$ and $a_{ij} = 0$, otherwise. The corresponding Laplacian matrix is given by $L = (l_{ij}) \in \mathbb{R}^{N \times N}$ with $l_{ii} = \sum_{j=1,\ j \neq i}^{N} a_{ij}$ and $l_{ij} = -a_{ij}$ for $i \neq j$. The links between the leader and the followers are characterized by matrix $B = diag(b_1, \ldots, b_N)$ where $b_i > 0$ if the leader state is available to follower i and where $b_i = 0$ otherwise.

In this paper, it is assumed that the communication topology among the N followers is undirected. It means that the adjacency matrix A is symmetric.

2.3 Problem statement

Consider a multi-agent system consisting of a leader (which could be virtual) labeled by 0, and N followers, labeled by $i \in \{1, \ldots, N\}$. The dynamics of the leader is given by the following chained-form nonholonomic system

$$\dot{\xi}_{1,0}(t) = \xi_{2,0}(t)$$
$$\dot{\xi}_{2,0}(t) = u_{1,0}(t)$$
$$\dot{\xi}_{3,0}(t) = \xi_{4,0}(t)\xi_{2,0}(t)$$
$$\dot{\xi}_{4,0}(t) = u_{2,0}(t) \tag{5}$$

where $\xi_0 = [\xi_{1,0}, \xi_{2,0}, \xi_{3,0}, \xi_{4,0}]^T \in \mathbb{R}^4$ is the leader state and $u_0 = [u_{1,0}, u_{2,0}]^T \in \mathbb{R}^2$ is the leader control input. The dynamics of the ith follower is as follows

$$\dot{\xi}_{1,i}(t) = \xi_{2,i}(t)$$
$$\dot{\xi}_{2,i}(t) = u_{1,i}(t) + d_{1,i}(t)$$
$$\dot{\xi}_{3,i}(t) = \xi_{4,i}(t)\xi_{2,i}(t)$$
$$\dot{\xi}_{4,i}(t) = u_{2,i}(t) + d_{2,i}(t) \tag{6}$$

where $\xi_i = [\xi_{1,i}, \xi_{2,i}, \xi_{3,i}, \xi_{4,i}]^T \in \mathbb{R}^4$ is the state and $u_i = [u_{1,i}, u_{2,i}]^T \in \mathbb{R}^2$ is control input of the ith follower. The unknown perturbation of the ith chained-form chained-form is given by $d_i = \left[d_{1,i}, d_{2,i}\right]^T \in \mathbb{R}^2$.

Hereafter, it is assumed that the following hypothesis hold to derive the proposed fixed-time controller.

Assumption 1 Graph $\mathcal{G}$ is undirected, fixed, connected and there is at least one strictly positive parameter b_i.

Assumption 2 The followers do not know the leader control input u_0. Nevertheless, its neighboring agents know its upper bounds $u_{1,0}$ and $u_{2,0}$, defined as follows

$$|u_{1,0}(t)| \leq u_{1,0}^{\max}$$
$$|u_{2,0}(t)| \leq u_{2,0}^{\max} \tag{7}$$

with $u_{1,0}^{\max}, u_{2,0}^{\max} \in \mathbb{R}^+$.

Assumption 3 For each follower, the perturbation $d_i(t)$ is unknown but it is bounded as follows

$$|d_{1,i}(t)| \leq d_{1,i}^{\max}$$
$$|d_{2,i}(t)| \leq d_{2,i}^{\max} \tag{8}$$

with $d_{1,i}^{\max}, d_{2,i}^{\max} \in \mathbb{R}^+$.

Assumption 4 It is assumed that the leader state $\xi_{2,0}$ satisfies the following condition

$$\xi_{2,0}(t) \neq 0, \qquad \forall t \in [T_m, T_M] \tag{9}$$

where $T_m < T_M$ are known positive constants.

Remark 2 Assumption 1 is conventional to deal with the leader–follower consensus problem. Hypotheses 2-3 are not restrictive since the bounds of perturbations and leader input can be estimated a priori for any system. Assumption 4 is relatively not restrictive since only the case $\xi_{2,0}(t) = 0, \forall t \geq 0$ is not considered in the proposed scheme. Therefore, the motion planner for the leader should take into account this constraint (to avoid any loss of controllability). Indeed, when $\xi_{2,0}(t) = 0$, we loose controllability of the leader dynamics since the state $\xi_{3,0}$ cannot be controlled. Since the followers track the leader, they also loose controllability in this case.

Remark 3 Note that there exist several tools to plan the trajectory for the leader. For instance, in [41], based on nonlinear programming and flatness properties, a constrained receding horizon planner is applied to design the state and the control input of the leader. One can easily extend this work while taking into account constraints (7) and (9).

Here, the purpose of this paper is to derive a decentralized observer-based controller u_i $(i = 1, \ldots, N)$ for each follower, based on available information, such that the leader–follower consensus problem is solved in a fixed-time, in spite of matched external perturbations. It means that there exists a positive constant T, selected as $T \leq T_M$, such that $\forall \xi_i(0), \forall i = 1, \ldots, N$,

$$\lim_{t \to T} \|\xi_i(t) - \xi_0(t)\| = 0$$
$$\|\xi_i(t) - \xi_0(t)\| = 0, \ \forall t \geq T \tag{10}$$

Remark 4 One can note that the settling time T does not depend on the initial states of the agents. It removes some limitations on existing finite-time consensus schemes for the application in decentralized architectures. The settling time can be prescribed according to some high-level policies (for instance in flexible manufacturing systems, when an operation should be performed before a deadline), or when cluster networks or switching communication topologies are considered [33].

3 Fixed-time leader–follower consensus protocol

For chained-form dynamics MAS under matched perturbations, we propose an observer-based consensus protocol to deal with the leader–follower consensus problem.

3.1 Decentralized fixed-time observer

To estimate the leader state in a prescribed time, decentralized observers are designed for each follower $i \in \{1, \ldots, N\}$. Indeed, the leader state is only available to its neighboring followers (see the definition of matrix B in Sect. 2.2). Let us introduce the following observer

$$\begin{aligned}
\dot{\hat{\xi}}_{1,i} &= \hat{\xi}_{2,i} \\
&\quad + \rho_1 \,\mathrm{sign}\left(\sum_{j=1}^{N} a_{ij}(\hat{\xi}_{1,j} - \hat{\xi}_{1,i}) + b_i(\xi_{1,0} - \hat{\xi}_{1,i})\right) \\
&\quad + \sigma_1 \left[\sum_{j=1}^{N} a_{ij}(\hat{\xi}_{1,j} - \hat{\xi}_{1,i}) + b_i(\xi_{1,0} - \hat{\xi}_{1,i})\right]^2 \\
\dot{\hat{\xi}}_{2,i} &= \rho_2 \,\mathrm{sign}\left(\sum_{j=1}^{N} a_{ij}(\hat{\xi}_{2,j} - \hat{\xi}_{2,i}) + b_i(\xi_{2,0} - \hat{\xi}_{2,i})\right) \\
&\quad + \sigma_2 \left[\sum_{j=1}^{N} a_{ij}(\hat{\xi}_{2,j} - \hat{\xi}_{2,i}) + b_i(\xi_{2,0} - \hat{\xi}_{2,i})\right]^2 \\
\dot{\hat{\xi}}_{3,i} &= \hat{\xi}_{4,i}\hat{\xi}_{2,i} \\
&\quad + \rho_3 \,\mathrm{sign}\left(\sum_{j=1}^{N} a_{ij}(\hat{\xi}_{3,j} - \hat{\xi}_{3,i}) + b_i(\xi_{3,0} - \hat{\xi}_{3,i})\right) \\
&\quad + \sigma_3 \left[\sum_{j=1}^{N} a_{ij}(\hat{\xi}_{3,j} - \hat{\xi}_{3,i}) + b_i(\xi_{3,0} - \hat{\xi}_{3,i})\right]^2
\end{aligned}$$

$$\dot{\hat{\xi}}_{4,i} = \rho_4 \operatorname{sign}\left(\sum_{j=1}^{N} a_{ij}(\hat{\xi}_{4,j} - \hat{\xi}_{4,i}) + b_i(\xi_{4,0} - \hat{\xi}_{4,i})\right) + \sigma_4 \left\lceil \sum_{j=1}^{N} a_{ij}(\hat{\xi}_{4,j} - \hat{\xi}_{4,i}) + b_i(\xi_{4,0} - \hat{\xi}_{4,i}) \right\rfloor^2 \tag{11}$$

where $\hat{\xi}_{k,i}$ ($k = \{1, \ldots, 4\}$) is the estimation of the leader state $\xi_{k,0}$ for the ith follower, ρ_k and σ_k are positive constants, which will be given hereafter.

The fixed-time stabilization of the estimation errors

$$\tilde{\xi}_{k,i} = \hat{\xi}_{k,i} - \xi_{k,0} \qquad (i = \{1, \ldots, N\},\ k = \{1, \ldots, 4\}) \tag{12}$$

is introduced in the following theorem.

Theorem 1 *Suppose that Assumptions 1–2 are satisfied. If the gains of the decentralized observer (11) verify*

$$\sigma_k = \frac{\epsilon\sqrt{N}}{(2\lambda_{\min}(L+B))^{\frac{3}{2}}}, \forall k = 1, \ldots, 4$$
$$\rho_1 = \epsilon\sqrt{\frac{\lambda_{\max}(L+B)}{2\lambda_{\min}(L+B)}}$$
$$\rho_2 = u_{1,0}^{\max} + \epsilon\sqrt{\frac{\lambda_{\max}(L+B)}{2\lambda_{\min}(L+B)}}$$
$$\rho_3 = \epsilon\sqrt{\frac{\lambda_{\max}(L+B)}{2\lambda_{\min}(L+B)}}$$
$$\rho_4 = u_{2,0}^{\max} + \epsilon\sqrt{\frac{\lambda_{\max}(L+B)}{2\lambda_{\min}(L+B)}} \tag{13}$$

with $\epsilon > \frac{2\pi}{T}$, then, for any initial condition, the estimation errors (12) converge to zero. An upper bound of the convergence time can be given as

$$T_o = \frac{2\pi}{\epsilon} \tag{14}$$

Proof Using (11), the observation error dynamics is given by

$$\dot{\tilde{\xi}}_{1,i} = \tilde{\xi}_{2,i} + \rho_1 \operatorname{sign}\left(\sum_{j=1}^{N} a_{ij}(\tilde{\xi}_{1,j} - \tilde{\xi}_{1,i}) - b_i\tilde{\xi}_{1,i}\right) + \sigma_1 \left\lceil \sum_{j=1}^{N} a_{ij}(\tilde{\xi}_{1,j} - \tilde{\xi}_{1,i}) - b_i\tilde{\xi}_{1,i} \right\rfloor^2$$

$$\dot{\tilde{\xi}}_{2,i} = \rho_2 \operatorname{sign}\left(\sum_{j=1}^{N} a_{ij}(\tilde{\xi}_{2,j} - \tilde{\xi}_{2,i}) - b_i\tilde{\xi}_{2,i}\right) + \sigma_2 \left\lceil \sum_{j=1}^{N} a_{ij}(\tilde{\xi}_{2,j} - \tilde{\xi}_{2,i}) - b_i\tilde{\xi}_{2,i} \right\rfloor^2 - u_{1,0}$$

$$\dot{\tilde{\xi}}_{3,i} = \rho_3 \operatorname{sign}\left(\sum_{j=1}^{N} a_{ij}(\tilde{\xi}_{3,j} - \tilde{\xi}_{3,i}) - b_i\tilde{\xi}_{3,i}\right) + \sigma_3 \left\lceil \sum_{j=1}^{N} a_{ij}(\tilde{\xi}_{3,j} - \tilde{\xi}_{3,i}) - b_i\tilde{\xi}_{3,i} \right\rfloor^2 + \hat{\xi}_{4,i}\hat{\xi}_{2,i} - \xi_{4,0}\xi_{2,0}$$

$$\dot{\tilde{\xi}}_{4,i} = \rho_4 \operatorname{sign}\left(\sum_{j=1}^{N} a_{ij}(\tilde{\xi}_{4,j} - \tilde{\xi}_{4,i}) - b_i\tilde{\xi}_{4,i}\right) + \sigma_4 \left\lceil \sum_{j=1}^{N} a_{ij}(\tilde{\xi}_{4,j} - \tilde{\xi}_{4,i}) - b_i\tilde{\xi}_{4,i} \right\rfloor^2 - u_{2,0} \tag{15}$$

Let us denote

$$\tilde{\xi}_k = \left[\tilde{\xi}_{k,1}, \ldots, \tilde{\xi}_{k,N}\right]^T \tag{16}$$

Then, for $\tilde{\xi}_1$, $\tilde{\xi}_2$ and $\tilde{\xi}_4$, one can obtain

$$\dot{\tilde{\xi}}_1 = \tilde{\xi}_2 - \rho_1 \operatorname{sign}\left((L+B)\tilde{\xi}_1\right) - \sigma_1 \left\lceil (L+B)\tilde{\xi}_1 \right\rfloor^2 \tag{17}$$

$$\dot{\tilde{\xi}}_2 = -\rho_2 \operatorname{sign}\left((L+B)\tilde{\xi}_2\right) - \sigma_2 \left\lceil (L+B)\tilde{\xi}_2 \right\rfloor^2 - \mathbf{1}u_{1,0} \tag{18}$$

$$\dot{\tilde{\xi}}_4 = -\rho_4 \operatorname{sign}\left((L+B)\tilde{\xi}_4\right) - \sigma_4 \left\lceil (L+B)\tilde{\xi}_4 \right\rfloor^2 - \mathbf{1}u_{2,0} \tag{19}$$

Then, we perform three steps of making proof:

– Let us first study subsystems (17)–(18).
  Consider the Lyapunov function for subsystem (18)

  $$V_1 = \frac{1}{2}\tilde{\xi}_2^T(L+B)\tilde{\xi}_2 \tag{20}$$

  Its time derivative is given by

  $$\dot{V}_1 = -\rho_2\tilde{\xi}_2^T(L+B)\operatorname{sign}\left((L+B)\tilde{\xi}_2\right) + \tilde{\xi}_2^T(L+B)\left(-\sigma_2\left\lceil (L+B)\tilde{\xi}_2 \right\rfloor^2 - \mathbf{1}u_{1,0}\right) \leq -(\rho_2 - u_{1,0}^{\max})\|(L+B)\tilde{\xi}_2\|_1$$

$$- \beta_2 N^{-\frac{1}{2}} (2\lambda_{\min}(L+B))^{\frac{3}{2}} V_1^{\frac{3}{2}}$$
$$\leq -\epsilon V_1^{\frac{1}{2}} - \epsilon V_1^{\frac{3}{2}}$$

Using Lemma 1 and (4), this inequality guarantees that $\tilde{\xi}_2$ is fixed-time stable at the origin with the settling time bounded by $\frac{\pi}{\epsilon}$.

After $\tilde{\xi}_2$ converges to zero (i.e., when $t \geq \frac{\pi}{\epsilon}$), the dynamics of $\tilde{\xi}_1$ reduces to

$$\dot{\tilde{\xi}}_1 = -\rho_1 \operatorname{sign}\left((L+B)\tilde{\xi}_1\right) - \sigma_1 \left\lceil (L+B)\tilde{\xi}_1 \right\rfloor^2 \quad (21)$$

Similarly to the previous step, the $\tilde{\xi}_1$ dynamics converges to zero. Indeed, consider the Lyapunov function for system (21)

$$V_2 = \frac{1}{2}\tilde{\xi}_1^T (L+B)\tilde{\xi}_1 \quad (22)$$

Its time derivative along (21) is given by

$$\dot{V}_2 = -\rho_1 \tilde{\xi}_1^T (L+B) \operatorname{sign}\left((L+B)\tilde{\xi}_1\right)$$
$$- \sigma_1 \tilde{\xi}_1^T (L+B) \left\lceil (L+B)\tilde{\xi}_1 \right\rfloor^2$$
$$\leq -\rho_1 \|(L+B)\tilde{\xi}_1\|_1$$
$$- \sigma_1 N^{-\frac{1}{2}} (2\lambda_{\min}(L+B))^{\frac{3}{2}} V_2^{\frac{3}{2}}$$
$$\leq -\epsilon V_2^{\frac{1}{2}} - \epsilon V_2^{\frac{3}{2}}$$

Hence, one can conclude that $\tilde{\xi}_2$ converges to zero and after that $\tilde{\xi}_1$ converges to zero in a fixed-time bounded by $2\frac{\pi}{\epsilon}$.

– Let us now study subsystem (19).
The time derivative of the following Lyapunov function

$$V_3 = \frac{1}{2}\tilde{\xi}_4^T (L+B)\tilde{\xi}_4 \quad (23)$$

along the solution of (19) results in

$$\dot{V}_3 = -\rho_4 \tilde{\xi}_4^T (L+B) \operatorname{sign}\left((L+B)\tilde{\xi}_4\right)$$
$$+ \tilde{\xi}_4^T (L+B) \left(-\sigma_4 \left\lceil (L+B)\tilde{\xi}_4 \right\rfloor^2 - \mathbf{1} u_{2,0}\right)$$
$$\leq -(\rho_4 - u_{2,0}^{\max}) \|(L+B)\tilde{\xi}_4\|_1$$
$$- \sigma_4 N^{-\frac{1}{2}} (2\lambda_{\min}(L+B))^{\frac{3}{2}} V_4^{\frac{3}{2}}$$
$$\leq -\epsilon V_3^{\frac{1}{2}} - \epsilon V_3^{\frac{3}{2}}$$

Using Lemma 1 and (4), this inequality guarantees that $\tilde{\xi}_4$ is fixed-time stable at the origin with the settling time bounded by $\frac{\pi}{\epsilon}$.

– After $\tilde{\xi}_2$ and $\tilde{\xi}_4$ converge to zero (i.e., when $t \geq \frac{\pi}{\epsilon}$), the dynamics of $\tilde{\xi}_3$ reduces to

$$\dot{\tilde{\xi}}_3 = -\rho_3 \operatorname{sign}\left((L+B)\tilde{\xi}_3\right)$$
$$- \sigma_3 \left\lceil (L+B)\tilde{\xi}_3 \right\rfloor^2 \quad (24)$$

Similarly to previously, considering the Lyapunov function $V_4 = \frac{1}{2}\tilde{\xi}_3^T (L+B)\tilde{\xi}_3$ yields in the fixed-time stable at the origin of $\tilde{\xi}_3$ with the settling time bounded by $2\frac{\pi}{\epsilon}$

Hence, one can conclude that the estimation errors (12) converge to zero in a fixed-time bounded by T_o. □

Remark 5 The conditions (13) on the observer gains are relatively strong since global information like N, $\lambda_{\min}(L+B)$ and $\lambda_{\max}(L+B)$ is required to guarantee the fixed-time convergence. Hence, each agent must have some global knowledge about the communication topology similarly to existing works on fixed-time consensus (see for instance [11,33,36,37]). One should highlight that such global information is needed to provide an explicit estimate of the settling time (which is a very interesting feature of the proposed scheme). It is worthy of noting that if a prescribed convergence time T_o is required, one can easily tune the observer gains according to (13) to estimate the leader state. Similarly to the work of [42], parameter ϵ should be tuned to obtain a good compromise between robustness against measurement noises (in the presence of measurement noises, only a neighborhood of the origin of the observation error system, which depends on the size of noise and ϵ, could be fixed-time stable) and sufficiently fast estimation. When the observer gains are selected, through conditions (13), the proposed scheme can be considered as a decentralized one since each neighboring agent only exchanges local information during the process.

3.2 Decentralized fixed-time controller

From Theorem 1, one can conclude that

$$\hat{\xi}_i = [\hat{\xi}_{1,i}, \hat{\xi}_{2,i}, \hat{\xi}_{3,i}, \hat{\xi}_{4,i}]^T = \xi_0$$

for all $t \geq T_o$. Hence, after time T_o, each follower is able to indirectly access to the state of the leader and uses the estimate $\hat{\xi}_i$ in the consensus protocol.

Let us define the tracking errors as follows

$$e_{k,i} = \xi_{k,i} - \hat{\xi}_{k,i} = \xi_{k,i} - \xi_{k,0} - \tilde{\xi}_{k,i} \quad (i = \{1, \ldots, N\},\ k = \{1, \ldots, 4\}) \tag{25}$$

From (5) to (6) and using Theorem 1, for each follower $i = \{1, \ldots, N\}$ and for all $t \geq T_o$, the tracking error dynamics reduces to

$$\begin{aligned} (\Sigma_1) \quad & \dot{e}_{1,i} = e_{2,i} \\ & \dot{e}_{2,i} = u_{1,i} + d_{1,i} - u_{1,0} \\ (\Sigma_2) \quad & \dot{e}_{3,i} = e_{4,i}\xi_{2,0} + (e_{4,i} + \xi_{4,0})e_{2,i} \\ & \dot{e}_{4,i} = u_{2,i} + d_{2,i} - u_{2,0} \end{aligned} \tag{26}$$

It is clear that dynamics (26) is divided into two coupled second-order subsystems (i.e., Σ_1 and Σ_2). To solve the fixed-time leader–follower consensus tracking control, for each follower $i = \{1, \ldots, N\}$, the following two steps are defined:

- Design $u_{1,i}$ such that the origin of subsystem Σ_1 is fixed-time stable with the settling time estimate $T_1 < T$.
- For $t \geq \max(T_1, T_m)$, design $u_{2,i}$ such that the origin of subsystem Σ_2 is fixed-time stable with the settling time estimate T. Note that for $t \geq T_1$, subsystem Σ_2 becomes

$$\begin{aligned} \dot{e}_{3,i} &= e_{4,i}\xi_{2,0} \\ \dot{e}_{4,i} &= u_{2,i} + d_{2,i} - u_{2,0} \end{aligned} \tag{27}$$

It is clear that for $t \geq T_1$, the two subsystems are decoupled.

The following theorem presents the control strategy which ensures the fixed-time leader–follower consensus of multi-agent systems with chained-form dynamics under matched external disturbances.

Theorem 2 *Consider the multi-agent system (5)–(6). Suppose that Assumptions 1–4 hold and the gains of the decentralized observer (11) satisfy (13). The fixed-time leader–follower consensus tracking control is solved using the decentralized controllers*

$$u_{1,i} = \begin{cases} 0, & \forall t < T_o \\ -\frac{\gamma_1 + 3\mu_1 e_{1,i}^2 + 2a_i}{2}\,\mathrm{sign}(s_{1,i}) - \lfloor \gamma_2 s_{1,i} + \mu_2 \lfloor s_{1,i} \rceil^3 \rceil^{\frac{1}{2}}, & \forall t \geq T_o \end{cases} \tag{28}$$

with the sliding surface

$$s_{1,i} = e_{2,i} + \lfloor \lfloor e_{2,i} \rceil^2 + \gamma_1 e_{1,i} + \mu_1 \lfloor e_{1,i} \rceil^3 \rceil^{\frac{1}{2}} \tag{29}$$

and

$$u_{2,i} = \begin{cases} 0, & \forall t < T_s \\ -\frac{1}{\hat{\xi}_{2,0}}\left(e_{4,i}\dot{\hat{\xi}}_{2,0} + \frac{\gamma_1 + 3\mu_1 e_{3,i}^2 + 2b_i}{2}\,\mathrm{sign}(s_{2,i})\right) - \frac{1}{\hat{\xi}_{2,0}}\lfloor \gamma_2 s_{2,i} + \mu_2 \lfloor s_{2,i} \rceil^3 \rceil^{\frac{1}{2}}, & \forall t \in [T_s, T_M] \\ -b_i\,\mathrm{sign}(e_{4,i}), & \forall t > T_M \end{cases} \tag{30}$$

with the sliding surface

$$s_{2,i} = e_{4,i}\xi_{2,0} + \lfloor \lfloor e_{4,i}\xi_{2,0} \rceil^2 + \gamma_1 e_{3,i} + \mu_1 \lfloor e_{3,i} \rceil^3 \rceil^{\frac{1}{2}} \tag{31}$$

where γ_1, γ_2, μ_1 and μ_2 are positive constants, a_i and b_i are positive constants given hereafter, and the switching time, which does not depend on the initial conditions of the system, is explicitly defined as

$$T_s = T_o + \frac{2}{\sqrt{\gamma_2}} + \frac{2}{\sqrt{\mu_2}} + \frac{2\sqrt{2}}{\sqrt{\gamma_1}} + \frac{2\sqrt{2}}{\sqrt{\mu_1}} \tag{32}$$

Proof We perform three steps of making proof. First, it will be shown that the origin of subsystem Σ_1 is fixed-time stable with the settling time estimate T_s under the control law (28)–(29). Then, the protocol (28)–(31) guarantees that the origin of subsystem Σ_2 is fixed-time stable with the settling time estimate T. At last, it will be proved that in spite of the presence of matched disturbances, the fixed-time leader–follower consensus control is solved.

- Let us first consider subsystem Σ_1. Following [27], let us consider the Lyapunov function candidate $V_5 = |s_{1,i}|$. Its derivative along the system trajectories is,

$$\dot{V}_5 = \dot{e}_{2,i}\,\mathrm{sign}(s_{1,i}) + \frac{|e_{2,i}|\dot{e}_{2,i}\,\mathrm{sign}(s_{1,i}) + \frac{\gamma_1 + 3\mu_1 e_{1,i}^2}{2} e_{2,i}\,\mathrm{sign}(s_{1,i})}{\left|\lfloor e_{2,i} \rceil^2 + \gamma_1 e_{1,i} + \mu_1 \lfloor e_{1,i} \rceil^3\right|^{\frac{1}{2}}} \tag{33}$$

Since

$$\lfloor \gamma_2 s_{1,i} + \mu_2 \lfloor s_{1,i} \rceil^3 \rceil^{\frac{1}{2}} \operatorname{sign}(s_{1,i}) = (\gamma_2 |s_{1,i}| + \mu_2 |s_{1,i}|^3)^{\frac{1}{2}}$$

one has

$$\begin{aligned}\dot{e}_{2,i} \operatorname{sign}(s_{1,i}) = (u_{1,i} + d_{1,i} - u_{1,0}) \operatorname{sign}(s_{1,i}) &= -\frac{\gamma_1 + 3\mu_1 e_{1,i}^2}{2} \\ &- (\gamma_2 |s_{1,i}| + \mu_2 |s_{1,i}|^3)^{\frac{1}{2}} \\ &- (a_i - (d_{1,i} - u_{1,0}) \operatorname{sign}(s_{1,i}))\end{aligned}$$

Setting

$$a_i \geq d_{1,i}^{\max} + u_{1,0}^{\max} \tag{34}$$

one can conclude that

$$\dot{V}_5 \leq -(\gamma_2 V_5 + \mu_2 V_5^3)^{\frac{1}{2}} \tag{35}$$

From Lemma 1, it is clear that $s_{1,i} = 0$ for all $t \geq T_o + \frac{2}{\sqrt{\gamma_2}} + \frac{2}{\sqrt{\mu_2}}$.
In sliding mode, i.e., $s_{1,i} = 0$, the dynamics becomes

$$\dot{e}_{1,i} = -\left\lfloor \frac{\gamma_1 e_{1,i} + \mu_1 \lfloor e_{1,i} \rceil^3}{2} \right\rceil^{\frac{1}{2}} \tag{36}$$

Let us consider the Lyapunov function candidate $V_6 = |e_{1,i}|$. Its derivative along the system trajectories is

$$\dot{V}_6 = -(\frac{\gamma_1}{2} V_6 + \frac{\mu_1}{2} V_6^3)^{\frac{1}{2}} \tag{37}$$

From Lemma 1, one can conclude that $e_{1,i} = 0$ for all $t \geq T_s$. One should notify that if $e_{1,i} = 0$ and $s_{1,i} = 0$, then $e_{2,i} = 0$.

- Let us now consider subsystem Σ_2 for $t \in [T_s, T_M]$. In this case, since $e_{1,i} = e_{2,i} = 0$, subsystem Σ_2 becomes (27). Note that Assumption 4 is introduced to avoid singularity problem in the controller design. Hence, let us set $\zeta_i = e_{4,i} \xi_{2,0}$. Dynamics (27) can be written as follows,

$$\begin{aligned}\dot{e}_{3,i} &= \zeta_i \\ \dot{\zeta}_i &= e_{4,i} \dot{\xi}_{2,0} + \xi_{2,0}(u_{2,i} + d_{2,i} - u_{2,0})\end{aligned} \tag{38}$$

Setting

$$b_i \geq d_{2,i}^{\max} + u_{2,0}^{\max} \tag{39}$$

and using the controller (30)–(31), similarly to the previous step, one can easily deduce that $e_{3,i} = 0$ for all

$$t \geq T = T_s + \frac{2}{\sqrt{\gamma_2}} + \frac{2}{\sqrt{\mu_2}} + \frac{2\sqrt{2}}{\sqrt{\gamma_1}} + \frac{2\sqrt{2}}{\sqrt{\mu_1}} \tag{40}$$

One should note that if $e_{3,i} = 0$ and $s_{2,i} = 0$, then $e_{4,i} = 0$ due to Assumption 4.

- The last step is to study subsystem Σ_2 for $t > T_M$. From subsystem Σ_2, one can obtain

$$\dot{e}_{4,i} = -b_i \operatorname{sign}(e_{4,i}) + d_{2,i} - u_{2,0} \tag{41}$$

The controller $u_{2,i}$ is used to reject the effect of uncertain terms $d_{2,i}$ and $u_{2,0}$. Let us consider the Lyapunov function candidate $V_7 = \frac{1}{2} e_{4,i}^2$. Its derivative along the system trajectories is

$$\begin{aligned}\dot{V}_7 &= -b_i |e_{4,i}| + e_{4,i}(d_{2,i} - u_{2,0}) \\ &\leq -(b_i + d_{2,i}^{\max} + u_{2,0}^{\max}) |e_{4,i}| \\ &\leq 0\end{aligned} \tag{42}$$

For the previous step, one has $e_{3,i}(T_M) = e_{4,i}(T_M) = 0$. Hence, one can conclude that for any initial condition, the tracking errors converge to zero in a finite-time bounded by T and remains there in spite of the presence of matched disturbances.

This concludes the proof. □

Remark 6 The design guidelines for our fixed-time controller are as follows:

- Select the parameter ϵ. Similarly to the work of [42], parameter ϵ should be tuned to obtain a good compromise between robustness against measurement noises and sufficiently fast estimation [T_o can be computed according to Eq. (14)].
- Design the observer gain parameters using Eq. (13).
- Select the settling time T. This parameter T should be tuned to obtain a good compromise between the control magnitudes and sufficiently fast convergence for the tracking errors.
- Select the switching time T_s as follows:

$$T_s = \frac{T - T_o}{2}$$

It enables to divide the time interval $[T_o, T_s]$ into two equal parts. This is useful for our two steps procedure described in Sect. 3.2 [i.e., fixed-time stabilization of system Σ_1 and system Σ_2 described in Eq. (26)].
- Knowing T_o and T_s, select the parameters γ_1, γ_2, μ_1 and μ_2 according to (32). A possible choice for these parameters is the following:

$$(\mu_1/2) = \mu_2 = (\gamma_1/2) = \gamma_2 = \frac{64}{(T_s - T_o)^2} \tag{43}$$

- Select the control parameters a_i and b_i which satisfy inequalities (34) and (39).

Remark 7 It is worth pointing out that Theorem 2 provides an explicit estimation of the settling time, i.e., (40), that can be prescribed a-priori. This is one of the key features of the results proposed in this paper, unlike the finite-time design in the existing literature [20]. In addition, Remark 6 provides a guideline for users to tune the control parameters. Note also that the fixed-time consensus problem for multiple nonholonomic agents has been dealt with in [11]. However, in [11], the leader is static and no uncertainty is considered in agent dynamics. To the best of the authors' knowledge, the fixed-time leader–follower consensus problem for multiple uncertain chained-form systems with dynamic leader is studied for the first time in this paper.

4 Simulation results

Here, the performances of the proposed observer-based leader–follower consensus controller are studied through numerical simulations.

Suppose that the MAS is composed of $N = 6$ followers labeled by $1-6$ and one leader labeled by 0. The nonlinear dynamics of the leader (resp. the followers) is given by (5) [resp. (6)]. Hereafter, the leader control input is set as $u_0 = [0.1\sin(t), -0.5\cos(0.5t)]$. Each follower is affected by the unknown perturbation $d_i = \left[0.2\sin(\xi_{1,i}), 0.3e^{-t})\right]$. Therefore, one can easily verify that Assumptions 2–3 are fulfilled with $u_{1,max} = 0.1$, $u_{2,max} = 0.5$, $d_{1,max} = 0.2$ and $d_{2,max} = 0.3$.

Figure 1 shows the communication topology. One can see that it is fixed and connected. It is characterized by the following Laplacian L and the matrix B which describes links between the leader and the followers given as follows

$$L = \begin{bmatrix} 2 & -1 & 0 & 0 & 0 & -1 \\ -1 & 2 & -1 & 0 & 0 & 0 \\ 0 & -1 & 3 & -1 & 0 & -1 \\ 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ -1 & 0 & -1 & 0 & -1 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} 0&0&0&0&0&0 \\ 0&1&0&0&0&0 \\ 0&0&0&0&0&0 \\ 0&0&0&0&0&0 \\ 0&0&0&0&1&0 \\ 0&0&0&0&0&0 \end{bmatrix}$$

From matrix B, it is clear that agents 1, 3, 4 and 6 do not have direct link with agent 0. Assumption 1 is satisfied. The initial leader state is set as $\xi_0(0) = [3, 2, 2, 0.5]^T$. From the control input of the leader, one can see that Assumption 4 is verified.

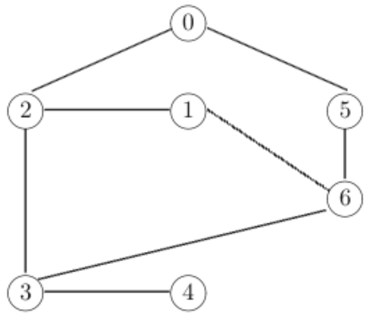

Fig. 1 Information flow among the leader and the followers

Here, the fixed-time consensus problem for the considered multiple chained-form systems is studied. It should be noted that contrary to finite-time schemes, the estimation of the settling time does not require the knowledge of the initial state, allowing a step-by-step design for the controller. It removes some limitations on existing finite-time consensus schemes for the application in decentralized architectures. The settling time can be prescribed according to some high-level policies or when cluster networks or switching communication topologies are considered. For each agent, a decentralized observer is designed to estimate the leader state in a fixed-time. To the best of the authors knowledge, even if the finite-time consensus problem for nonholonomic systems have been already studied, an estimation of the settling time, independently of the initial conditions, has not yet been given for multiple chained-form systems.

The parameter ϵ is set as $\epsilon = 2\pi$. Therefore, using (13), the parameters of the decentralized observer are selected as $\rho_1 = \rho_3 = 21$, $\rho_2 = 21.1$, $\rho_4 = 21.5$ and $\sigma_k = 52$, $\forall k = 1, \ldots, 4$. The settling time is chosen as $T = 9s$. From (43), inequalities (34) and (39), the parameters of the decentralized controller are chosen as follows: $\gamma_1 = 8$, $\mu_1 = 8$, $\gamma_2 = 4$, $\mu_2 = 4$, $a_i = 0.3$ and $b_i = 0.8$.

Using Theorem 1, the decentralized observer (11) guarantees the stabilization of the estimation errors (12) to the origin in a finite-time bounded by $T_o = 1s$. Figure 2 shows that the decentralized observers accurately reconstruct the leader state for each agent before T_o where different initial conditions of the observer for each agent are selected). Seeing the observation error dynamics given in Eq. (15), it is clear that the evolution of $\tilde{\xi}_{1,i}$ (resp. $\tilde{\xi}_{2,i}$) will look like the evolution of $\tilde{\xi}_{3,i}$ (resp. $\tilde{\xi}_{4,i}$). Therefore, Fig. 2a (resp. Fig. 2b) looks

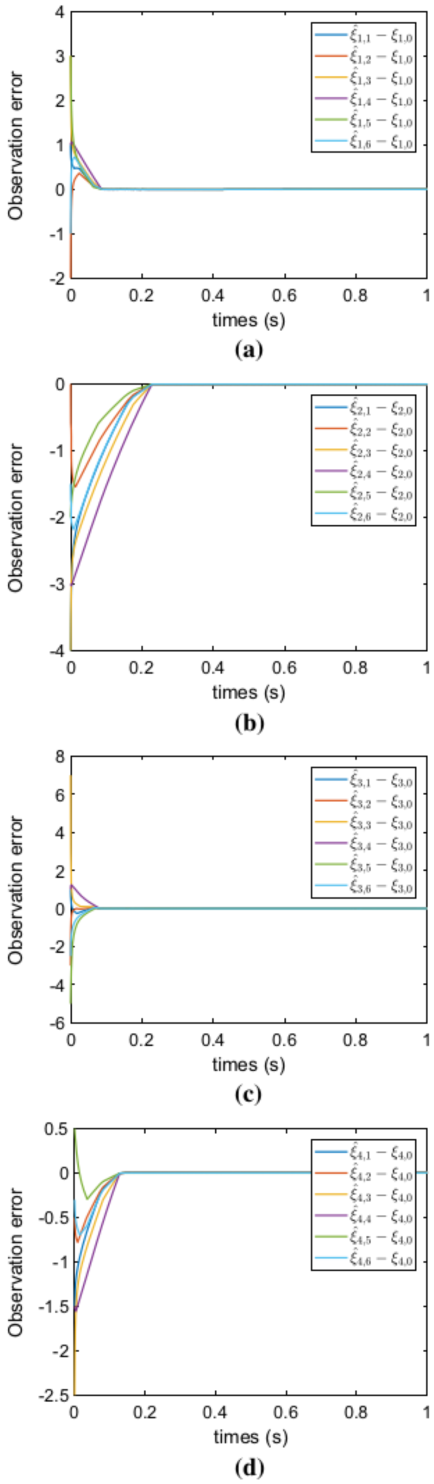

Fig. 2 Evolution of the estimation errors (12) for each follower

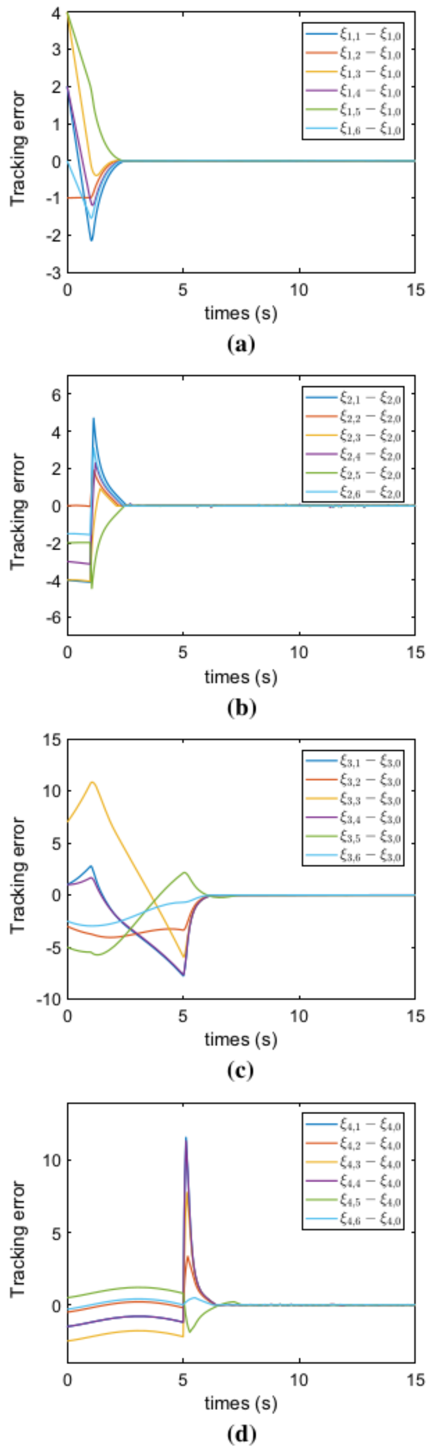

Fig. 3 Evolution of the tracking errors between each agent and the leader

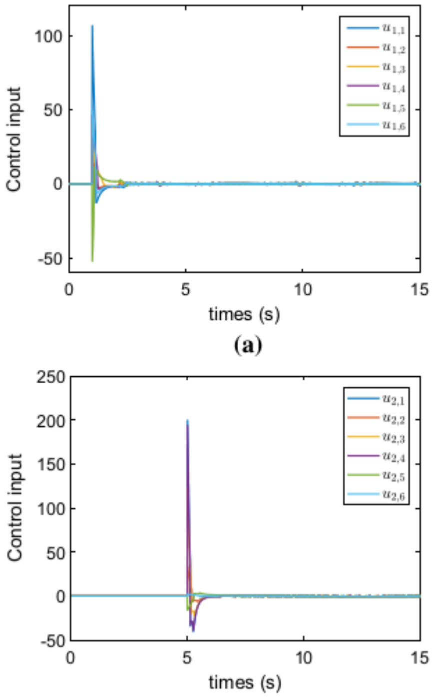

Fig. 4 Control input for each agent

like Fig. 2c (resp. Fig. 2d). Contrary to [11,33], the proposed fixed-time observer solves the communication loop problem due to the dependance of the control inputs of the followers on the inputs of its neighbors. The switching time in (30), which does not depend on the initial conditions of the system, can be computed using (14), i.e., $T_s = 5s$. Hence, the origin of the closed-loop system is globally finite-time stable (contrary to many existing finite-time controllers where only semi-global finite-time stability is guaranteed). Furthermore, since T_s does not depend on the initial states of agents, the proposed protocol is decentralized.

Here, an upper bound of the settling time, independently of the initial conditions, can be estimated using (32), i.e., $T = 9s$ from Theorem 2. The tracking errors are depicted in Fig. 3. One can see that the tracking errors between each follower and the leader converge to zero before T. From Fig. 3, one can conclude that using the proposed controller, the leader–follower consensus is achieved in a prescribed time. The control inputs for each agent are shown in Fig. 4. One can note that the magnitude of control inputs may be large during the transients to achieve a fast convergence of the sliding surfaces given by the different steps of the consensus protocol. Hence, the control parameters should be adjusted to obtain a good compromise between magnitude of the control inputs and settling time.

Based on the simulation results, one can see that the proposed observer-based controller achieves the fixed-time leader–follower consensus for MAS with chained-form dynamics in spite of the presence of matched perturbations.

5 Conclusion

In this paper, the fixed-time consensus problem of multiple chained-form systems under matched perturbations has been considered. Thanks to the proposed decentralized observers, the leader state has been estimated in a fixed-time. A decentralized observer-based control protocol has been proposed for each agent to solve the leader–follower consensus problem in a fixed-time. In future works, it will be possible to investigate fixed-time leader–follower consensus control strategies for MAS with chained-form dynamics in consideration of unmatched perturbations, noise measurement and directed graphs.

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Compliance with ethical standards

Conflict interest The authors declare that they have no conflict of interest.

Human and animal rights The research work does not involve any human participants and/or animals.

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